PHYSICAL REVIEW E VOLUME 48, NUMBER 2 AUGUST 1993

Prototype for memory effects in the time evolution of damage

Constantino Tsallis, Francisco Tamarit, and André M. C. de Souza*
Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290 Rio de Janeiro, Rio de Janeiro, Brazil
(Received 11 February 1993)

We introduce a one-dimensional cellular automaton as a prototype for memory effects on damage. The associated Hamming distance as a function of time correctly mimics complex dynamical systems and, for different values of the external parameters, gradually varies between a noiselike behavior and a plateaulike one.

PACS number(s): 05.60.+w, 02.50.−r, 05.45.+b, 64.60.Ht

A large number of complex dynamical systems present relevant quantities which behave, as functions of time, in a more or less noiselike manner. Many electronic, optic, acoustic-device, and meteorological phenomena, as well as various theoretical models, exhibit such behavior. Among these models we can include the discrete sandpile model [1], which presents self-organized criticality, as well as other granular systems (e.g., clogging in granular

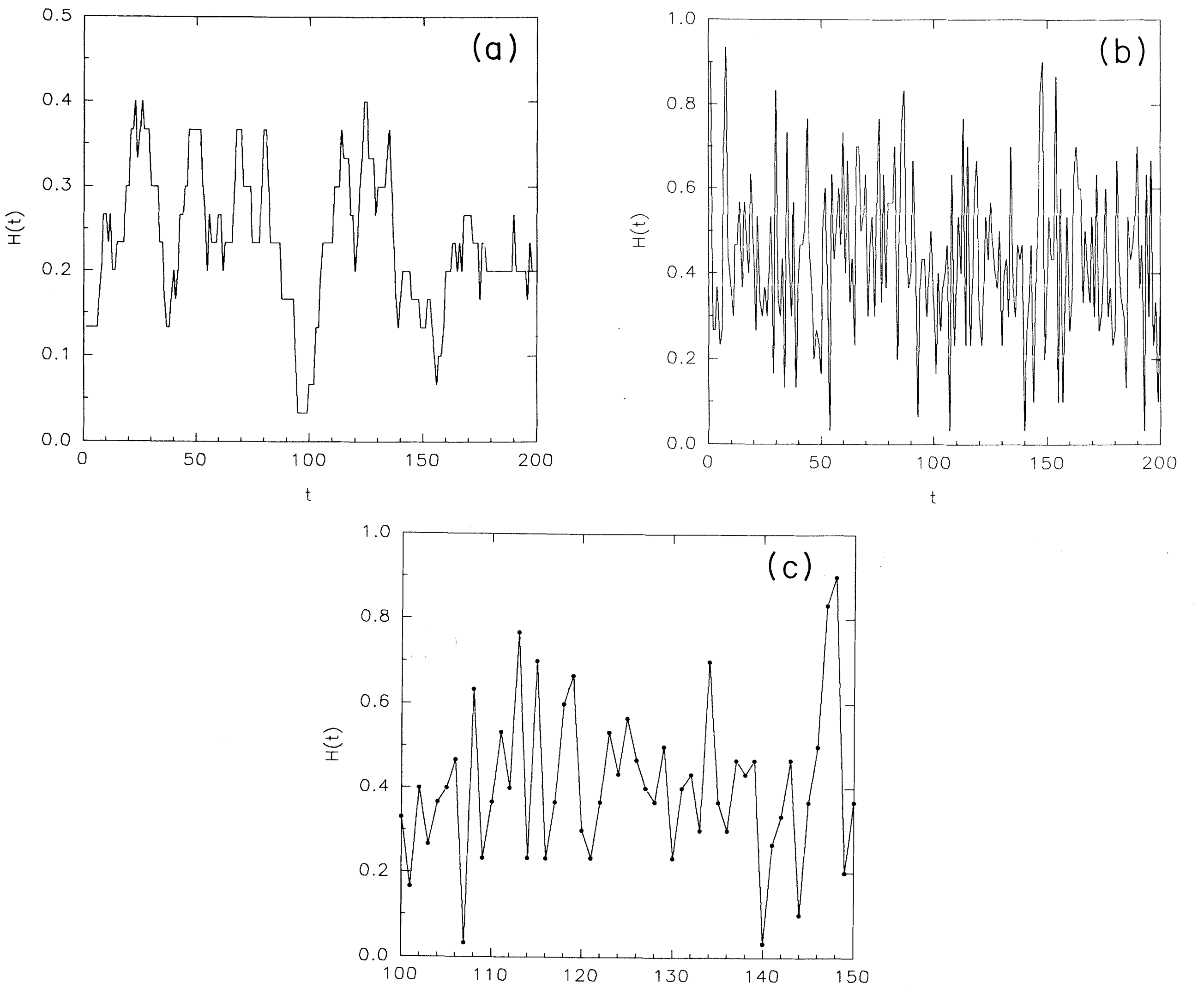

FIG. 1. Time evolution of the Hamming distance for $p=0.9$ and $L=30$: (a) $J=2$ (plateaux exist); (b) $J=30$ (plateaux do not exist); (c) enlargement of a typical region of (b).

material flowing in a pipe [2] or a continuous sandpile model [3]). Various relevant quantities can be studied in such models. One of them is the Hamming distance, which characterizes a damage introduced in the system. This type of situation is well illustrated on the discrete sandpile model: the time evolution of a conveniently defined Hamming distance has been recently studied by Erzan and Sinha [4]. It presents a noiselike dependence on time, expect for the (surprising) presence of abrupt jumps between plateaux (see Fig. 1 of [4]), which indicate the existence of some type of memory. The purpose of the present work is to propose a simple prototype—a one-dimensional deterministic cellular automaton—which can exhibit a similar type of memory effect, in a more or less distinct manner which can be tuned through the external parameters.

Let us assume a semi-infinite linear chain of sites ($i=0,1,2,\ldots$) occupied by binary random variables $\{S_i\}$ ($S_i=0,1\ \forall i$). We consider two equivalent replicas of the system ($\{S_i^A\}$ and $\{S_i^B\}$) constructed as follows. We conventionally assume $S_0^\alpha=1$ ($\alpha=A,B$) and then set $S_{i+1}^\alpha=S_i^\alpha$ with probability p [hence different from S_i^α with probability $(1-p)$]. Although the value p is shared by both strips, the random sequences used to generate the actual strips are *different*. We now focus on a window of length L and define the following Hamming distance:

$$H(t)=\frac{1}{L}\sum_{i=i_0}^{i_0+L-1}|S_i^A-S_i^B| \ , \tag{1}$$

where $i_0\equiv Jt$, J being a fixed positive integer and time $t=0,1,2,\ldots$. Let us stress that the present model can be seen as a cellular automaton. Indeed, once the semi-infinite chain configuration is frozen, we look at a window whose width always is L (i.e., L is the size of the cellular automaton). The time evolution comes from the fact that we start reading the L sites from a point i_0 of the semi-infinite strip which linearly increases with time.

$H(t)$ will clearly fluctuate, and the fluctuations are expected to decrease for increasing L. In Fig. 1 we present two typical cases corresponding to $L=30$ (chosen, in this illustration, to coincide with the *linear* size of the sample used in [4]) and $p=0.9$; cases (a) and (b) respectively correspond to small J ($J=2$) and large J ($J=30$). We verify that our Fig. 1(a) is qualitatively similar to Fig. 1 of [4], whereas Fig. 1(b) just exhibits trivial fluctuations. In Fig. 1(c) we have enlarged a typical region of Fig. 1(b) in order to show that it does *not* look like a rescaled version of Fig. 1(a).

Let us now quantitatively describe the H-vs-t graph. If $H(t+1)\neq H(t)$, there is no plateau at time t ($\tau=0$); if $H(t+2)\neq H(t+1)=H(t)$, we shall say that there is a $\tau=1$ plateau; if $H(t+3)\neq H(t+2)=H(t+1)=H(t)$, we shall say that the plateau is a $\tau=2$ one, etc. For fixed (p,J,L), $H(t)$ yields a distribution $P(\tau)$ associated with the plateau $[\sum_{\tau=0}^{\infty}P(\tau)=1]$; $M(p,J,L)\equiv 1-P(0)$ is the probability of having finite-size plateaux and, in some sense, plays the role of an order parameter. We now present the average $M(p,J,L)$ as obtained through simu-

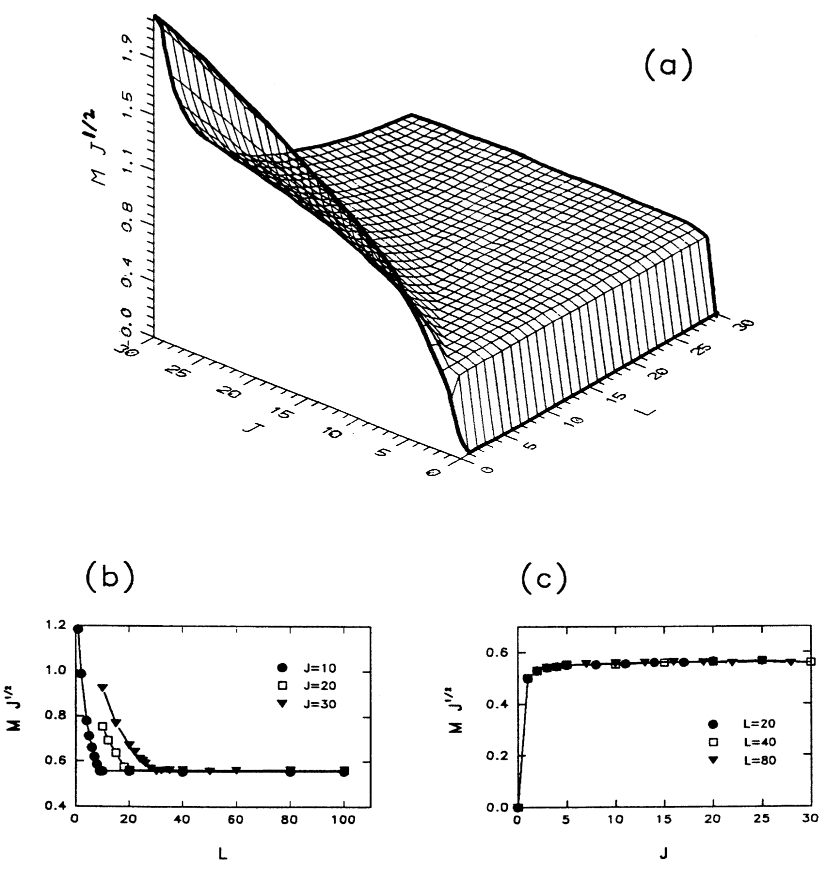

FIG. 2. (J,L) dependence of $M\sqrt{J}$ for $p=0.5$. (a) full diagram; (b) fixed L cuts; (c) fixed J cuts.

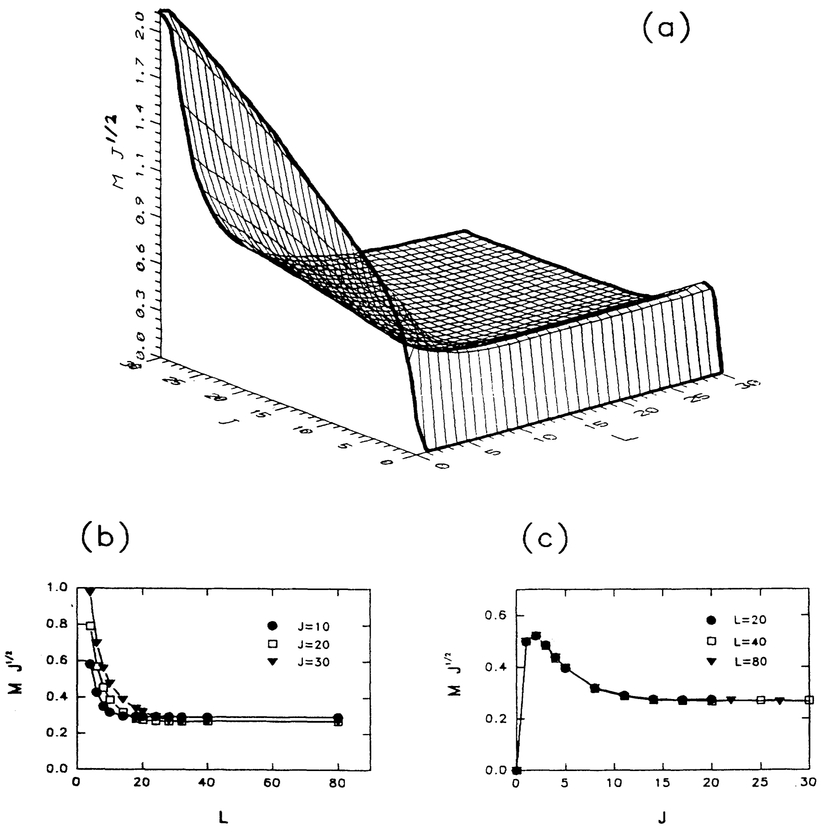

FIG. 3. (J,L) dependence of $M\sqrt{J}$ for $p=0.9$. (a) full diagram; (b) fixed L cuts; (c) fixed J cuts.

lations in which we have performed about 1000 experiments, each of which run up to $t=1000$ (or $t=5000$ in some cases): see Figs. 2 and 3 for typical examples.

We see that representing $M\sqrt{J}$ (instead of M) yields a convenient data collapse. For fixed p, four different regimes can be identified, namely, (i) $J\simeq L\simeq 1$; (ii) $J>L>1$, (iii) $J<J^*(p)$ and $L>1$, and (iv) $J^*(p)<J<L$,

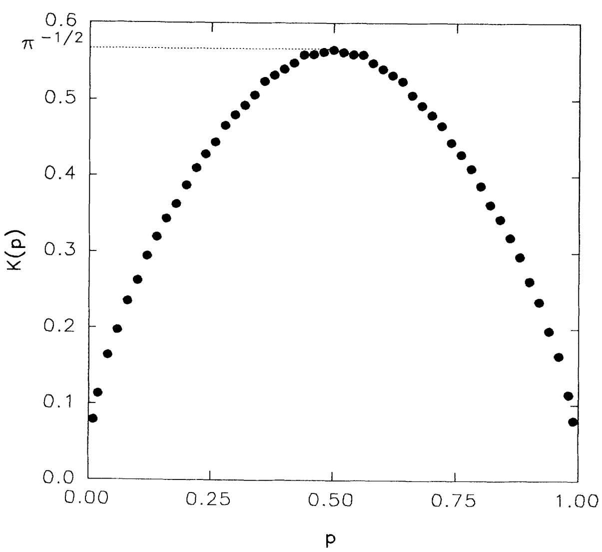

FIG. 4. p dependence of $K\equiv\lim_{J\to\infty}M(p,J,\infty)\sqrt{J}$.

where the crossover value $J^*(p)$ satisfies $J^*(p)=J^*(1-p)$ [e.g., $J^*(0.5)\simeq 2$, $J^*(0.9)=J^*(0.1)\simeq 10$, and $J^*(1)=J^*(0)=\infty$]. Memory disappears (i.e., $M\to 0$) for any value of L, and $0<p<1$ whenever $J\to\infty$. In the thermodynamic limit $L\to\infty$, only two regimes subsist, namely, regime (iii) $[J<J^*(p)]$, where no scaling exists for M, and regime (iv) $[J>J^*(p)]$, where $M\propto 1/\sqrt{J}$. As intuitively expected, $J^*(p)$ monotonously increases when p increases from 0.5 to 1; indeed, when p approaches unity, memory persists for increasingly larger values of J. In regime (iv), all transients have disappeared, and in Fig. 4 we show the p dependence of $K(p)\equiv\lim_{J\to\infty}\lim_{L\to\infty}M(p,J,L)\sqrt{J}$.

The analytical discussion of the $p=\frac{1}{2}$ case (full randomness) is relatively simple in the limit $L\to\infty$. It suffices, for a jump of size J, to consider the J initial sites (yielding 2^J different configurations) and the J final sites (yielding 2^J different configurations); indeed, the configurations of the $(L-2J)$ internal sites do not contribute for M. So, the analysis of these $2^J 2^J=4^J$ configurations leads to

$$M(0.5,J,\infty)=\frac{\sum_{l=0}^{J}\binom{J}{l}^2}{4^J}\,, \tag{2}$$

hence

$$M(0.5,J,\infty)=\frac{\binom{2J}{J}}{4^J}\,. \tag{3}$$

The use of Stirling's formula immediately yields, in the $J \to \infty$ limit,

$$M(0.5,J,\infty) \sim \frac{1}{\sqrt{\pi J}} , \tag{4}$$

hence

$$K(0.5) \equiv \lim_{J \to \infty} M(0.5,J,\infty)\sqrt{J} = \frac{1}{\sqrt{\pi}} \tag{5}$$

thus confirming the numerical result indicated in Fig. 4.

Let us conclude by recalling that a possibly large class of systems exhibiting memory effects in the time evolution of a damage might belong to to the same "universality class" as that of the prototype we have herein introduced. In any case, it could be so for the discrete sandpile model recently studied by Erzan and Sinha. In fact, it would be interesting to study whether these two models do or do not belong, in some sense, to the same class. As further developments, it could be interesting to study the momenta of $P(\tau)$ [e.g., $\langle \tau \rangle \equiv \sum_{\tau=0}^{\infty} \tau P(\tau)$], as well as d-dimensional versions of the present model.

We acknowledge interesting remarks from A. Erzan and H. J. Hermann. F. T. and A. M. C. S. have been supported, respectively, by CNPq and CAPES (Brazilian agencies).

---

*Permanent address: Departamento de Física, Universidade Federal de Sergipe, Sergipe, Brazil.

[1] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987); Phys. Rev. A **38**, 364 (1988); K. Wiesenfeld, C. Tang, and P. Bak, J. Stat. Phys. **54**, 1141 (1989).

[2] T. Pöschel, Hochstleistungsrechenzentrum Report No. HLRZ **67**, 1992 (unpublished).

[3] J. Lee and H. J. Herrmann, Hochstleistungsrechenzentrum Report No. HLRZ **44**, 1992 (unpublished).

[4] A. Erzan and S. Sinha, Phys. Rev. Lett. **66**, 2750 (1991).